New Exact Solutions and Complex Wave Excitations for the (2+1)-Dimensional Asymmetric Nizhnik-Novikov-Veselov System

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Starting from an improved mapping approach and a linear variable separation approach, new families of variable separated solutions (including solitary wave solutions, periodic wave solutions and rational function solutions) with arbitrary functions for the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov (ANNV) system are derived. Based on the derived solutions, we obtain some special complex wave excitations.

Key words: Improved Mapping Approach; ANNV System; Exact Solutions; Complex Wave Excitations.
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1. Introduction

Mathematical modelling of physical systems often leads to nonlinear partial differential equations [1 – 3]. Exact solutions of such equations play an important role in nonlinear science, especially in nonlinear physical science, since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications [4 – 8]. Seeking the exact solutions of partial differential equations has long been an interesting and hot topic in nonlinear mathematical physics [9 – 14]. Previously, much efforts have been focused on simple localized excitations, such as solitoffs, dromions, rings, lumps, breathers, instantons, peakons, compactons, localized chaotic, fractal patterns [15 – 20], and some authors have investigated the propagation of solitons in the plane [21, 22]. Now an important and interesting question is what evolutional behaviours will happen if the solitons propagate on a background wave. In this paper as a concrete example we consider the asymmetric Nizhnik-Novikov-Veselov (ANNV) system

$$Q_t + Q_{xxx} - 3v_xQ - 3vQ_x = 0, \quad Q_x - v_y = 0. \quad (1)$$

Equation (1) was deduced by Boiti et al. [23] and it can also be obtained from the Kadomtsev-Petviashvili equation [24]. In [25], Ma et al. have derived the peakon and the fractal localized excitations of the equation.

2. New Exact Solutions to the ANNV System

As is known, to search for the exact solutions of a nonlinear physical model, we can apply different approaches. One of the most efficient methods to find soliton excitations of a physical model is the so-called improved mapping approach. The basic idea of the algorithm is as follows. For a given nonlinear partial differential equation (NPDE) with the independent variables $x = (x_0 = t, x_1, x_2, \cdots, x_m)$ and the dependent variable u in the form

$$P(u, u_t, u_{x_i}, u_{x_ix_j}, \cdots) = 0, \quad (2)$$

where P is in general a polynomial function of its arguments, and the subscripts denote the partial derivatives, the solution may be assumed to be in the form [25]

$$u = A(x) + \sum_{i=1}^{n} \left\{ B_i(x)\phi^i[q(x)] + \frac{C_i}{\phi^i[q(x)]} + D_i(x)\phi^{i-1}[q(x)]\sqrt{\sigma + \phi^2[q(x)]} + \frac{E_i(x)}{\phi^{i-1}[q(x)]\sqrt{\sigma + \phi^2[q(x)]}} \right\} \quad (3)$$

with

$$\phi' = \sigma + \phi^2, \quad (4)$$

where $A(x)$, $B_i(x)$, $C_i(x)$, $D_i(x)$, $E_i(x)$, $q(x)$ are functions of the indicated argument to be determined, σ is an arbitrary constant, and the prime denotes ϕ differentiation with respect to q. To determine u explicitly, one substitutes (3) and (4) into the given NPDE and collects the coefficients of the polynomials of ϕ, then eliminates each coefficient to derive a set of partial differential equations for A, B_i, C_i, $D_i(x)$, $E_i(x)$, and q, and solves the system of partial differential equations to obtain A, B_i, C_i, $D_i(x)$, $E_i(x)$, and q. Finally, as (4) is known to possess the general solutions

$$\phi = \begin{cases} -\sqrt{-\sigma}\tanh(\sqrt{-\sigma}q), & \sigma < 0, \\ \sqrt{\sigma}\tan(\sqrt{\sigma}q), & \sigma > 0, \\ \dfrac{-1}{q}, & \sigma = 0, \end{cases} \tag{5}$$

and substituting A, B_i, C_i, $D_i(x)$, $E_i(x)$, q and (5) into (3), one obtains the exact solutions to the given NPDE (the coth-type and cot-type solutions are neglected here). Now we apply the improved mapping approach to (1). By the balancing procedure, the ansatz (3) becomes

$$\begin{aligned} Q &= f + g\phi + h\phi^2 + a\sqrt{\sigma+\phi^2} + b\phi\sqrt{\sigma+\phi^2} \\ &\quad + \frac{c}{\phi} + \frac{d}{\phi^2} + \frac{m}{\sqrt{\sigma+\phi^2}} + \frac{n}{\phi\sqrt{\sigma+\phi^2}}, \\ v &= F + G\phi + H\phi^2 + A\sqrt{\sigma+\phi^2} + B\phi\sqrt{\sigma+\phi^2} \\ &\quad + \frac{C}{\phi} + \frac{D}{\phi^2} + \frac{M}{\sqrt{\sigma+\phi^2}} + \frac{N}{\phi\sqrt{\sigma+\phi^2}}, \end{aligned} \tag{6}$$

where f, g, h, a, b, c, d, m, n, F, G, H, A, B, C, D, M, N and q are functions of (x,y,t) to be determined. Substituting (6) and (4) into (1) and collecting coefficients of the polynomials in ϕ, then setting each coefficient to zero, we have

$$\begin{aligned} f &= 2\sigma\frac{q_{xx}q_y - q_xq_{xy}}{q_xq_y}, \quad g = -\frac{3q_xq_{xy} + q_{xx}q_y}{q_x}, \\ h &= q_xq_y, \quad a = \frac{8q_{xx}q_y + 3q_xq_{xy}}{q_x}, \quad b = -q_xq_y, \\ c &= -\frac{3q_xq_{xy} - 4q_xq_{yy}\sigma}{2q_xq_y}, \quad d = 2q_xq_y, \\ m &= \frac{3q_xq_{xy} + 8q_yq_{xx}}{2q_xq_y}, \quad n = -2q_xq_y, \\ F &= -\left[q_{xx}^2q_y^2 + q_x^2q_{xy}^2 + q_xq_yq_{xx}q_{xy} \right. \\ &\quad \left. - q_x^4q_y^2 + q_x^3q_y\sigma - q_xq_y^2q_t\right]\left[q_x^2q_y^2\right]^{-1}, \\ G &= \frac{3q_xq_{xy} + q_{xx}q_y}{q_y}, \quad H = q_x^2, \\ A &= -\frac{3q_xq_{xy} + 8q_{xx}q_y}{q_y}, \quad B = -q_x^2, \\ C &= -\frac{2q_xq_{xy} - 3q_xq_{yy}\sigma}{3q_xq_y}, \quad D = 2q_{xx}q_y, \\ M &= \frac{2q_xq_{xy} + 3q_yq_{xx}}{3q_xq_y}, \quad N = -2q_{xx}q_y. \end{aligned} \tag{7}$$

Here the function q has the variable separated form

$$q = \chi(x,t) + \varphi(y), \tag{8}$$

where $\chi(x,t)$ and $\varphi(y)$ are two arbitrary functions of the indicated arguments. Based on the solutions of (4) reported by many previous authors [26–28], one thus obtains an explicit solution of (1).

Case 1. For $\sigma = -1$, we can derive the following solitary wave solutions of (1):

$$\begin{aligned} Q_1 = -\chi_x\varphi_y\Big[&1 - \tanh(\chi+\varphi)^2 \\ &- \tanh(\chi+\varphi)\sqrt{\tanh(\chi+\varphi)^2 - 1}\Big], \end{aligned} \tag{9}$$

$$\begin{aligned} v_1 = \frac{1}{3}&\frac{\chi_{xxx} + \chi_x^3 + \chi_t}{\chi_x} \\ &- \chi_{xx}\left[\tanh(\chi+\varphi) + \sqrt{\tanh(\chi+\varphi)^2 - 1}\right] \\ &- \chi_x^2\Big[1 - \tanh(\chi+\varphi)^2 \\ &\quad - \tanh(\chi+\varphi)\sqrt{\tanh(\chi+\varphi)^2 - 1}\Big], \end{aligned} \tag{10}$$

with two arbitrary functions being $\chi(x,t)$ and $\varphi(y)$.

Case 2. For $\sigma = 1$, we obtain the following periodic wave solutions of (1):

$$\begin{aligned} Q_2 = \chi_x\varphi_y\Big[&1 + \tan(\chi+\varphi)^2 \\ &- \tan(\chi+\varphi)\sqrt{\tan(\chi+\varphi)^2 + 1}\Big], \end{aligned} \tag{11}$$

$$\begin{aligned} v_2 = -\frac{1}{3}&\frac{\chi_{xxx} - \chi_x^3 - \chi_t}{\chi_x} \\ &+ \chi_{xx}\left[\tan(\chi+\varphi) - \sqrt{\tan(\chi+\varphi)^2 + 1}\right] \\ &+ \chi_x^2\Big[1 + \tan(\chi+\varphi)^2 \\ &\quad - \tan(\chi+\varphi)\sqrt{\tan(\chi+\varphi)^2 + 1}\Big]. \end{aligned} \tag{12}$$

Again $\chi(x,t)$ and $\varphi(y)$ are two arbitrary functions.

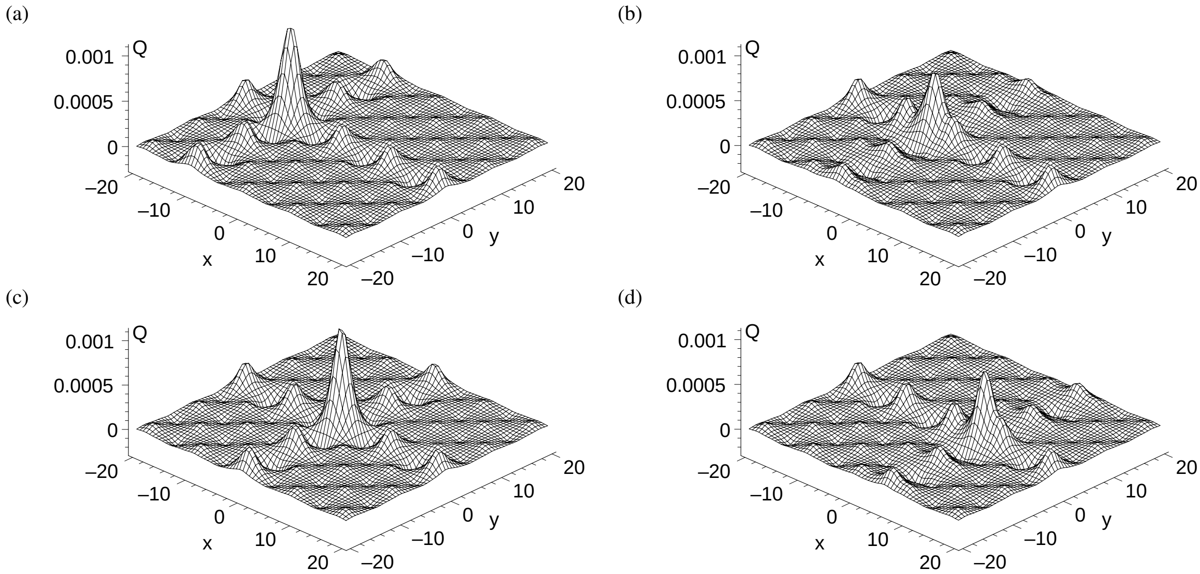

Fig. 1. Time dependence of the profile of a single-dromion on a periodic wave background for the solution Q expressed by (15) under the condition (17) at different times: (a) $t=-6$; (b) $t=-2$; (c) $t=0$; (d) $t=3$.

Case 3. For $\sigma=0$, we can derive the following variable separated solution of (1):

$$Q_3 = 2\frac{\chi_x\varphi_y}{(\chi+\varphi)^2}, \tag{13}$$

$$v_3 = \frac{1}{3}\frac{\chi_{xxx}+\chi_t}{\chi_x} - \frac{2\chi_{xx}}{\chi+\varphi} + \frac{2\chi_x^2}{(\chi+\varphi)^2}, \tag{14}$$

with two arbitrary functions being $\chi(x,t)$ and $\varphi(y)$.

3. Complex Wave Excitations in the ANNV System

Owing to the arbitrariness of the functions $\chi(x,t)$ and $\varphi(y)$ included in the above solutions, the physical quantities Q and v may possess rich localized structures. For simplicity in the following discussion, we merely analyze the rational function solution Q_3 expressed by (13) and rewrite it in a simple form, namely

$$Q = Q_3 = 2\frac{\chi_x\varphi_y}{(\chi+\varphi)^2}. \tag{15}$$

3.1. The Evolution of a Single-Dromion in a Periodic Wave Background

We now focus our attention on the intriguing evolution of solitons on a background wave for the solution Q. In order to reveal a complex wave excitation in the ANNV system, here we choose χ and/or φ to be a combined function of solitary wave and periodic wave, respectively, i. e.,

$$\begin{aligned}\chi(x,t) &= 1+\sum_{i=1}^{m} a_i L_i(x,t) + \sum_{i=1}^{m} b_i P_i(x,t),\\ \varphi(y) &= 1+\sum_{j=1}^{n} a_j l_j(y) + \sum_{j=1}^{n} b_j p_j(y),\end{aligned} \tag{16}$$

where $P_i(x,t), p_j(y)$ are some suitable periodic wave functions, such as Jacobi function and Airy function, $L_i(x,t), l_j(y)$ are some localized functions like tanh and sech functions, while a_i and b_i are arbitrary real constants. When a_i become larger the amplitude of the dromion will be correspondingly larger, and similarly, the amplitude of the periodic wave will increase with b_i increasing since a_i and b_i represent the amplitudes of the solitary wave and the background wave, respectively. The amplitude of the background wave will tend to zero in the limit case $a_i \gg b_i$, otherwise the amplitude of the dromion will tend to zero as $b_i \gg a_i$.

Figure 1 shows a time evolution profile of a single-dromion moving on a periodic wave background for the physical quantity Q of (15) with

$$\begin{aligned}\chi &= 1+0.08\tanh(0.6x-t)+0.02\,\mathrm{sn}(0.8x,0.3),\\ \varphi &= 1+0.08\tanh(0.6y)+0.02\,\mathrm{sn}(0.8y,0.3).\end{aligned} \tag{17}$$

From Fig. 1 we can see that the dromion moves in a

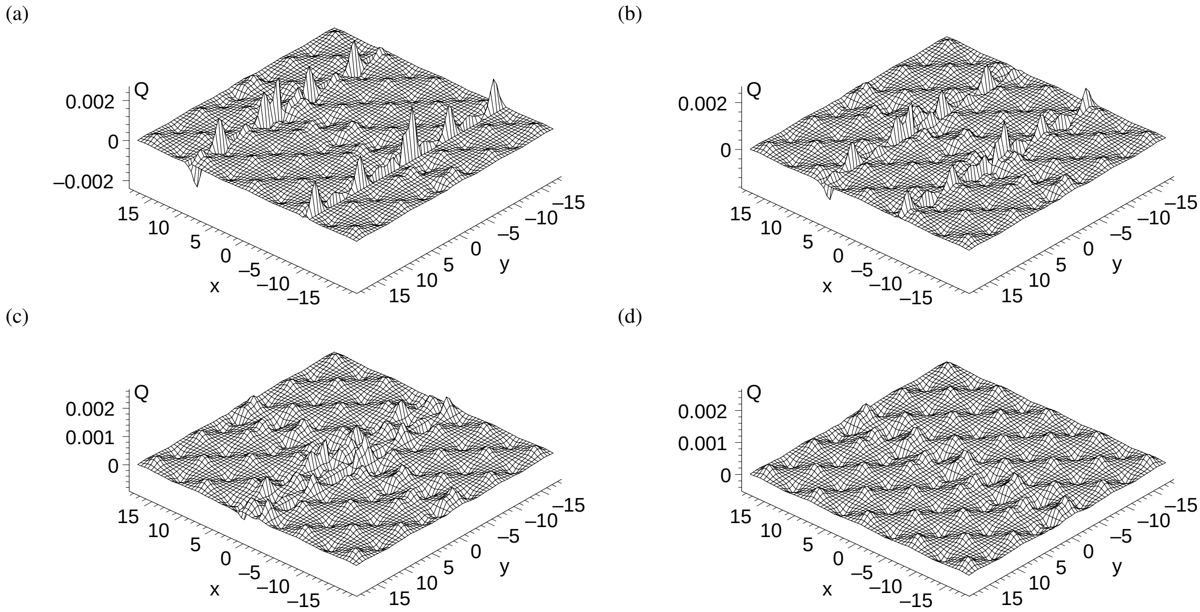

Fig. 2. Plot of the annihilation of solitons in a periodic wave background for the solution Q from (15) under the condition (18) at different times: (a) $t=-45$; (b) $t=-20$; (c) $t=-1$; (d) $t=10$.

periodic wave background, and the amplitude of the dromion changes with time because of the superposition of the solitary wave and the background wave.

3.2. The Annihilation of Solitons in a Periodic Wave Background

Just like other particles solitons can also be annihilated in some appropriate conditions. For example, we choose χ and φ in solution (15) to be

$$\begin{aligned}\chi &= 1+0.08\,\mathrm{sech}(0.5x^2+t)+0.03\mathrm{sn}(0.8x,0.4),\\ \varphi &= 1+0.08\,\mathrm{sech}(0.5y^2)+0.05\mathrm{sn}(0.8y,0.4),\end{aligned} \quad (18)$$

where sn is still the Jacobi sine function. We observe the annihilation of solitons on a periodic wave background for the physical quantity Q of (15) under the condition (18) as shown in Figure 2. From Fig. 2, we find that the amplitude and shape of the solitons become smaller and smaller after their interactions, and finally, they reduce to zero.

4. Summary and Discussion

In summary, via an improved mapping approach and a linear variable separation approach, the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov system is solved. Abundant localized coherent soliton structures of the solution Q of (15), like dromions, peakons, breathers, instantons, can be revealed by choosing appropriate arbitrary functions. Except for the above localized excitation, we have found a new type of complex wave excitations, i. e., solitons moving on a periodic wave background.

Actually, the complex wave excitations are usually encountered in nature, such as a billow (a large ocean wave) going with some small surface waves, an optical communicating signal wave concomitant with certain noise background waves [29]. Some specific evolutional behaviours of the complex wave excitations in other nonlinear dynamical physical systems are recently reported [29, 30]. However, the interactions between the single-dromion and the periodic wave background and the annihilation of solitons on a periodic wave background for the (2+1)-dimensional ANNV system were little reported in the previous literature. We hope that these complex wave excitations related to the Jacobi sine function would be helpful to some applications in reality, and the results derived in the paper may be useful to thoroughly understand the novel complex wave excitations.

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